2825. Proposed by the late L. G. WELD.

A ball, having a coefficient of resilience α, strikes a rigid plane surface, inclined at an angle θ from the horizontal, after falling through a height h. What is the distance from the first to the second point of impact with the plane?

2826. Proposed by ALBERT A. BENNETT, University of Texas.

As a standard form for a square non-singular symmetric matrix under certain transformations, may be taken the form in which only the elements in the secondary diagonal are different from zero, and each of these is equal to unity. Analogously, as a standard form for a square non-singular skew-symmetric matrix (and hence incidentally of even order), may be taken the form in which only the elements of the secondary diagonal are different from zero, while the half of these which are towards the upper right-hand corner are each minus one, and the remaining half towards the lower left-hand corner, are each plus one. Denote both of these standard matrices by N.

Give simple parallel proofs that if M be given as non-singular and symmetric or non-symmetric as the case may be, a matrix P exists such that, with the usual notation

$$M = PNP'.$$

2827. Proposed by B. F. FINKEL, Drury College.

Find the equation of the envelope of the system of circles inscribed in a triangle having a given base and a given altitude.

2828. Proposed by T. M. BLAKSLEE, Ames, Iowa.

On page 72 of R. B. Hayward's *The Algebra of Coplanar Vectors and Trigonometry* occurs the sentence: "It will be a good exercise for the student to show that cos $(90°/7) = \frac{1}{2}\sqrt{x_1}$, where x_1 is the greatest root of the equation,

$$x^3 - 7x^2 + 14x - 7 = 0."$$

(1) Do not merely verify but deduce the equation and find x_1. (2) Deduce the x-equation (x_1, x_2, x_3, x_4, the roots) such that its greatest root x_1 gives $\cos(90°/9) = \cos 10° = \frac{1}{2}\sqrt{x_1}$. (3) Of what angles are $\frac{1}{2}\sqrt{x_1}, \cdots \frac{1}{2}\sqrt{x_4}$, in (2), the cosines? Develop a method of writing out at once $\cos(nv)$ in terms of powers of $\cos v$ if these are given for $(n-1)v$ and $(n-2)v$. The same for $\sin(nv)$. (4) Use the results of (2) and (3) to find the number of degrees in a radian. Hence, find π from radian instead of radian from π as is usual.

SOLUTIONS OF PROBLEMS.

411 (Algebra) [1914, 121; 1919, 268, 459]. Proposed by V. M. SPUNAR, Chicago, Ill.

Determine $x_1, x_2, x_3 \cdots x_p$ from the equations:

$$\begin{aligned}
x_1 + x_2 + x_3 + \cdots + x_p &= a_0,\\
b_1x_1 + b_2x_2 + b_3x_3 + \cdots + b_px_p &= a_1,\\
b_1^2x_1 + b_2^2x_2 + b_3^2x_3 + \cdots + b_p^2x_p &= a_2,\\
\cdots\cdots\cdots\cdots\cdots\cdots &\cdots\\
b_1^{p-1}x_1 + b_2^{p-1}x_2 + b_3^{p-1}x_3 + \cdots + b_p^{p-1}x_p &= a_{p-1}.
\end{aligned}$$

A solution has been sent in by P. J. DA CUNHA, University of Lisbon, Portugal, in which the analysis is very much the same as that previously printed, but he also considers the case when some of the b's are equal. For example, if $p = 3$ and $b_1 = b_2 \gtrless b_3$, we must have

$$\begin{vmatrix} 1 & 1 & a_0 \\ b_1 & b_3 & a_1 \\ b_1^2 & b_3^2 & a_2 \end{vmatrix} = 0.$$

If this condition is satisfied, the expressions for x_1 and x_2 in the general solution take indeterminate forms, and there will be an infinite number of solutions given by

$$x_1 + x_2 = \frac{a_0b_3 - a_1}{b_3 - b_1}, \qquad x_3 = \frac{a_0b_1 - a_1}{b_1 - b_3}.$$